THE AGENCY COST OF ALTERNATIVE DEBT INSTRUMENTS

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Sloan School of Management
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In this paper we show how to adapt the traditional contingent claims valuation techniques to correctly value the firm and its liabilities in the presence of agency costs. This enables us to measure the significance of the agency costs as a function of the quantity of debt outstanding and as a function of the design of the debt contract: with this we can determine the relative benefits of alternative contract designs. Our work makes a contribution to the recent debate about how much debt a corporation can prudently assume. It is possible to measure the agency advantages of the new debt instruments for a given firm, and therefore to determine for which firms the advantages are significant and for which firms they are not.

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It is now commonly recognized that a firm's capital structure can affect its value through the incentives that are created for the equity holders in favor of one or another investment and operating policy. A place has therefore been created for a positive theory of capital structure. Missing, however, from the literature on agency costs in finance have been models that enable us to measure the effects of capital structure on the value of the firm's assets. In this paper we show how contingent claims models can be used to measure and compare the agency costs of different forms of debt--fixed rate and indexed. The model can be used to determine the optimal indexing structure and the optimal parameters of the debt contract. The case of index linked debt that we study in this paper is a commodity bond.

The standard contingent claims pricing model abstract from the very factors that must be at the centerpiece of a positive theory of capital structure. For example, there already exist many contingent claims models for

pricing commodity linked debt instruments under a variety of assumptions, beginning with Schwartz (1982) and including Ingersoll (1982) Carr (1987), Kemna (1987) and Rajan (1988); but in each of these models the stochastic process governing the value of the firm is exogenously specified and is unaffected by any agency problems. This makes it impossible that different capital structures could induce the management to pursue different investment programs and therefore induce different stochastic processes for the value of the firm. Consequently these contingent claims models cannot help us understand which firms ought to issue commodity linked bonds, nor why such bonds exist at all.

This failure is not accidental: contingent claims models of commodity bonds are all extensions of Merton's (1974) model for the pricing of risky debt. As Merton points out, the Modigliani-Miller theorem obtains in his model: the value of the firm is independent of the amount and the type of leverage. On the other hand, the traditional agency models in which the Modigliani-Miller theorem does not obtain cannot generally be put to practical use. In order to allow a careful modelling of the specific strategic relations that are analyzed in detail, the parameters of the models are either so simplified that it is impossible to associate them with measurable parameters of a real world case, or else the models simply abstract from certain critical factors--such as a robust measure of price risk--that must be incorporated into any real application. For example, although we now understand that sinking funds, dividend restrictions, and other bond covenants help to resolve the conflict of interest between bondholders and equity, we do not yet have any operative models with which to determine the optimal parameters of these very covenants.

In order to apply the contingent claims techniques to a setting in which agency problems are central some adaptation of the commonly used techniques is necessary. The value of the firm cannot itself be an exogenously specified stochastic process, but must instead be an endogenous function of an underlying state variable. We use the traditional contingent claims model to determine the value of alternative operating strategies, and--based upon this valuation model--we use the traditional agency and game theoretic techniques to determine endogenously the firm's choice of operating strategies and therefore the realized stochastic process describing the value of the firm and its liabilities. Different assumed financial structures will yield different operating strategies and therefore different realized stochastic processes for the value of the firm from which the actual values of the assumed liabilities are calculated.

Our work makes a contribution to the debate that has arisen in recent years about how much debt a corporation can prudently assume--see for example Jensen (1989) and Lowenstein (1985). In this paper we present the first analytically rigorous model in which it is possible to demonstrate the increased debt capacity created by new financial instruments which lower the agency costs of debt. Moreover, our model does not provide a blanket case in favor of these new instruments: rather, it allows us to measure the agency advantages of the new debt instruments for a given firm, and therefore to determine for which firms the advantages are significant and for which firms they are not.

To illustrate this new application of contingent claims analysis to the problem of calculating the agency costs of alternative debt instruments we extend the Brennan and Schwartz (1985) valuation model of a natural resource

extraction firm to incorporate the incentive properties of the firm's capital structure.

1. A Contingent Claims Valuation of the Firm in the Presence of Agency Costs

Brennan and Schwartz analyze a firm that owns a mine with a commodity inventory, Q. When the mine is open the commodity is extracted at a constant annual rate, q, and at a constant real average annual cost, a. When the mine is closed a constant real annual maintenance cost, m, is incurred. At any point in time the mine can be closed at a real cost k_1 and reopened at a real cost k_2. The mine can also be costlessly abandoned.

Several crucial assumptions are made on the stochastic structure of the commodity price. First, the real spot price of the commodity, s, is determined in a competitive market and follows the exogenously determined process

$$ds = \mu s\ dt + \sigma s\ dz, \tag{1}$$

where dz is the increment to a standard Gauss-Wiener process; σ, the instantaneous standard deviation of the spot price, is assumed to be known and constant; and μ is the instantaneous drift in the real price. Second, it is assumed that there is a traded futures contract on the commodity. Then, following Ross (1978), if the convenience yield on the commodity is a constant proportion of the spot price, $\kappa(s) = \kappa s$, and if there exists a known constant real interest rate, r, the real price of a futures contract maturing in τ periods is given by $f(s,\tau) = se^{(r-\kappa)\tau}$.

The market value of the mine, v, is a function of the current commodity price, s, of the inventory, Q, of whether the mine is currently closed or open, j=1,2, and of the optimal operating policy, ϕ, $v \equiv v(s,Q;j,\phi)$. An

operating policy is described by three critical commodity prices: s_0, the price at which the mine is abandoned if it is already closed, s_1, the price at which the mine is closed if it was previously open, and s_2, the price at which the mine is opened if it was previously closed, $\phi = (s_0, s_1, s_2)$.[1] Applying Ito's lemma of stochastic calculus the instantaneous change in the value of the mine is given by $dv = v_s ds + v_Q dQ + \frac{1}{2} v_{ss}(ds)^2$. The cash flow from the mine is $q(s-a)(j-1) - m(2-j)$. Using an arbitrage argument similar to Black-Scholes the differential equation governing the value of the closed mine is

$$\tfrac{1}{2}\sigma^2 s^2 v_{ss}(s,Q;1) + (r-\kappa)s v_s(s,Q;1) - m - rv(s,Q;1) = 0, \tag{2}$$

and the open mine is

$$\tfrac{1}{2}\sigma^2 s^2 v_{ss}(s,Q;2) + (r-\kappa)s v_s(s,Q;2) - q v_Q(s,Q;2) + q(s-a) - rv(s,Q;2) = 0. \tag{3}$$

The first best operating policy $\phi^{FB} = (s_0^{FB}, s_1^{FB}, s_2^{FB})$ is characterized by the following first order conditions:

$$v_s(s_0^{FB},Q;1) = 0, \tag{4}$$

$$v_s(s_1^{FB},Q;2) = \begin{cases} v_s(s_1^{FB},Q;1) & \text{if } v(s_1^{FB},Q;1) - k_1 \geq 0 \\ 0 & \text{if } v(s_1^{FB},Q;1) - k_1 < 0, \end{cases} \tag{5}$$

$$v_s(s_2^{FB},Q;1) = v_s(s_2^{FB},Q;2). \tag{6}$$

These three equations serve as boundary conditions with which we can solve simultaneously for the first best value of the mine and the first best operating policy, v^{FB} and ϕ^{FB}. Four additional boundary conditions are used:

$$v(s,0;j) = 0 \tag{7}$$

$$v(s_0^{FB},Q;1) = 0, \tag{8}$$

$$v(s_1^{FB},Q;2) = \max\{v(s_1^{FB},Q;1) - k_1, 0\}, \tag{9}$$

$$v(s_2^{FB},Q;1) = v(s_2^{FB},Q;2) - k_2. \tag{10}$$

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[1] The extraction rate for an open mine is assumed constant at q.

To our knowledge there is no closed-form solution to equations (2) and (3) subject to boundary conditions (4)-(10). It is, however, possible to solve this system of equations using numerical methods as we have done for the hypothetical mine described in Table 1: The first best operating policy is given in Table 2 and the first best value of the mine is displayed in Table 4.

The first best solution is not generally attainable if the firm is financed in part with debt. To analyze the second best value of the mine and the value of the debt and equity we incorporate the firm's financial structure into the simultaneous solution of the optimal operating policy. We assume that the mine is financed in part with a bond requiring annual payments $\psi(s)$ during the life of the bond, $t \leq \tau$.

The market value of the equity prior to maturity of the bond, e, is then a function of the current commodity price, s, of the inventory, Q, of whether the mine is currently closed or open, j=1,2, of the modified operating policy, ϕ', and of the outstanding bond payment structure, ψ, $e \equiv e(s,Q;j,\phi',\psi)$. The modified operating policy acknowledges the right of the equity owners to default on the bond and is described by three critical commodity prices, $\phi' = (s_d, s_1, s_2)$: s_d is the price at which the equity owners default, while s_1 and s_2 are, as before, the prices at which the mine is closed or opened, respectively. Again applying Ito's lemma the instantaneous change in the value of the equity is given by $de = e_s ds - qe_Q + \frac{1}{2}e_{ss}(ds)^2$. The cash flow from the equity is $q(s-a)(j-1) - m(2-j) - \psi(s)$. The differential equation governing the value of the equity when the mine is closed is:

$$\frac{1}{2}\sigma^2 s^2 e_{ss}(s,Q;1) + (r-\kappa)se_s(s,Q;1) - qe_Q(s,Q;1) - m - \psi(s) - re(s,Q;1) = 0, \tag{11}$$

and when the mine is open is:

$$\tfrac{1}{2}\sigma^2 s^2 e_{ss}(s,Q;2) + (r-\kappa)se_s(s,Q;2) - qe_Q(s,Q;2)$$
$$+ q(s-a) - \psi(s) - re(s,Q;2) = 0. \tag{12}$$

The boundary conditions for this pair of differential equations are written in terms of the parameters of the modified operating policy, $\phi'^{\psi}=(s_d^{\psi},s_1^{\psi},s_2^{\psi})$, which maximizes the value of the equity given the terms of the outstanding bond:

$$e_s(s_d^{\psi},Q;1) = 0, \tag{13}$$

$$e_s(s_1^{\psi},Q;2) = \begin{cases} e_s(s_1^{\psi},Q;1) & \text{if } e(s_1^{\psi},Q;1) - k_1 \geq 0 \\ 0 & \text{if } e(s_1^{\psi},Q;1) - k_1 < 0, \end{cases} \tag{14}$$

$$e_s(s_2^{\psi},Q;1) = e_s(s_2^{\psi},Q;2), \tag{15}$$

along with the additional boundary conditions:

$$e(s,0;j) = 0, \tag{16}$$

$$e(s_d^{\psi},Q;1) = 0, \tag{17}$$

$$e(s_1^{\psi},Q;2) = \max\ \{e(s_1^{\psi},Q;1)-k_1,\ 0\}, \tag{18}$$

$$e(s_2^{\psi},Q;1) = e(s_2^{\psi},Q;2) - k_2. \tag{19}$$

Again it is necessary to solve simultaneously for the value of the equity and for the parameters of the optimal operating policy, e^{ψ} and $\phi'^{\psi}=(s_d^{\psi},s_1^{\psi},s_2^{\psi})$.

It is important to note that in general the operating policy chosen to maximize the value of the equity claim will not be identical with the first best operating policy, $(s_d^{\psi},s_1^{\psi},s_2^{\psi}) \neq (s_0^{FB},s_1^{FB},s_2^{FB})$. Consequently the value of the levered firm is less than the first best value calculated earlier, $v^{\psi} < v^{FB}$. We wish to emphasize that it is possible to identify the effect of the financial structure on the operating policy and therefore to measure the agency costs associated with a particular financial structure. Only then is it possible to correctly value the mine and the associated liabilities.

To determine the value of the levered firm it is necessary to solve the pair of differential equations (2) and (3) with boundary conditions based upon the operating policy that is optimal for the equity owners:

$$v(s,0;j) = 0, \tag{20}$$

$$v(s_d^{\not{v}},Q;1) = \alpha\, v^{FB}(s_d,Q;1), \tag{21}$$

$$v(s_1^{\not{v}},Q;2) = \max\{v(s_1^{\not{v}},Q;1) - k_1, 0\}, \tag{22}$$

$$v(s_2^{\not{v}},Q;1) = v(s_2^{\not{v}},Q;2) - k_2. \tag{23}$$

The value for the levered mine calculated using this system of equations is denoted $v^{\not{v}}$.

Boundary condition (21) requires some comment. Upon default the firm is put to the bondholder. The case in which the firm is subsequently operated according to the first best operating policy is equivalent to setting $\alpha=1$. Another more general case incorporates the possibilities that either (i) there are costs of financial distress associated with bankruptcy, or (ii) the bondholder cannot operate the firm and must reorganize it with a similar debt/equity structure--thereby reproducing the agency problem. This case is described by setting $\alpha \in [0,1)$. The parameter α then serves as a parameter measuring the significance of the costs of financial distress, and as α approaches zero these agency costs increase.

The value for the outstanding bond is the difference between the total value of mine and the value of the equity:

$$b^{\not{v}} = v^{\not{v}} - e^{\not{v}}. \tag{24}$$

To illustrate the model we calculated values for $v^{\not{v}}$, $e^{\not{v}}$ and $b^{\not{v}}$ for a hypothetical example. The input parameters for our example are given in Table 1. The equity owners' optimal operating policy, $\phi^{\not{v}}=(s_d^{\not{v}},s_1^{\not{v}},s_2^{\not{v}})$ is displayed in

Table 2 and contrasted with the first best operating policy. The values for the levered firm, levered equity, and for the bond are displayed in Table 3.

[Insert Tables 1, 2 and 3 Here]

Since the operating policy chosen to maximize the value of the equity is not the first best operating policy the value of the levered firm is less than the first best value of the firm, $v^{\ell} < v^{FB}$, the difference being the agency cost of debt. In Table 4 the values for v^{ℓ} are compared against the values for v^{FB} for the sample parameters described above. The size of the agency costs of debt is also calculated in Table 4.

[Insert Table 4 Here]

2. The Optimality of Commodity Linked Debt

Throughout the 1980's a large number of firms floated a new type of debt instrument with obligations linked to the price of the commodity.[2] We believe that agency costs were an important factor in this financial innovation. In this section we first present the agency argument in favor of commodity linked debt, and then we use the model to price and compare fixed and commodity linked bonds as financing instruments.

Consider a mining firm with the rights to a particular territory and with expertise in the efficient extraction of this commodity. The initial owner of the firm does not have enough capital with which to develop the mine and it

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[2] For example, in 1985 the Dutch venture capital corporation Oranje Nassau, a company with substantial investments in offshore oil drilling in the North Sea, issued bonds denominated in 1000 guilders but tied as well to the price of 10½ barrels of North Sea oil: at maturity the bond would be redeemed at the face value plus the amount by which the settlement price exceeds the face value (Kamma, 1987). In 1988 the Magma Corporation, the largest copper producer in the US, issued $200 million in notes with quarterly interest payments that would vary between 12 and 21% as the per annum average copper price ranged between $0.80 and $2 per pound (Priovolos and Duncan, 1989). In one of the most recent commodity linked financings the copper corporation Mexicana de Cobre borrowed $210 million from a syndicate of banks led by Banque Paribas using a more complicated packaging of contracts. Revenue from the sales of the copper to Société Génerale de Belgique at market prices are to be deposited in an escrow account used to payoff the loan. A parallel swap of the copper price was negotiated directly between Banque Paribas and Mexicana de Cobre.

will be necessary to bring in some outside capital. This can be done using either a new issue of equity or with debt. The management of the firm is presumed to have information about some important determinants of the mine's profitability that the potential pool of outside investors do not have. For example, the management of the firm may be informed of the exact costs of extraction from the mine while the outside investors would only know the average costs predominating in the industry as a whole. This inside information creates an adverse selection problem for the sale of new equity as established in Myers and Majluf (1984) and can make floating new equity a prohibitively costly financing choice. This leads the firm to prefer debt as its source of outside capital.

The debt contract, however, induces a variety of agency problems of its own. For example, under certain circumstances--in particular when the firm finds itself close to bankruptcy--the existence of debt may give the equity holders of the firm an incentive to forego certain valuable investments or an incentive to choose exclusively risky projects, or to otherwise choose a suboptimal investment program for the firm--see Myers (1977). The agency cost of debt can be especially severe for firms in the mining and petroleum industries. The value of the firm's inventory of the commodity in the ground is sometimes the largest asset on the firm's balance sheet. The market value of the firm therefore fluctuates significantly with the price of the commodity, and if the firm has a large outstanding debt obligation it may be driven close to bankruptcy by the movement of the commodity price regardless of how efficiently the management of the firm has operated its mines.

A commodity linked bond combines the advantages of both the equity and the debt instruments, while avoiding in part the agency costs associated with

each. While the promised payments on fixed rate bonds or other more traditional forms of debt are independent of the many exogenous variables determining the fortunes of the firm, the promised payment on the commodity linked bond, in contrast, rises and falls with the price of the commodity and therefore with the firm's ability to pay.[3] At the same time, since the commodity price is an observable and contractable exogenous variable, the commodity linked bond avoids some of the dangers of inside information about the firm's competitive advantage and therefore the adverse selection problems associated with outside equity sales.[4]

Our argument is a direct application of the results of the optimal contract design literature. For example, in Holmström (1979) it is shown that an agent's incentive payment should be variable in any observable parameter that provides information on the agent's performance. If we view the equity contract as an incentive contract, and if we imagine that the event of bankruptcy is meant to penalize the equity owner for poor performance, then Holmström's results establish that the equity owner should be thrown into bankruptcy only when the observed low level of profits was probably related to poor performance: if the low prevailing commodity price was the cause of low profits, then the firm should not be thrown into bankruptcy. It is precisely

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[3] Lessard (1977) makes a case for the use of commodity linked financing to LDC's dependent on one or two commodities for export revenue. Commodity linked debt would allow the country to shift a portion of the price risk to investors in the developed countries. Lessard's argument assumes that the firm's equity cannot readily be traded among investors from the developed countries: commodity linked bonds are proposed as an instrument that can help fill out the set of cross border securities with which risk can be efficiently shifted and shared among investors from different countries.

Our analysis demonstrates the applicability of Lessard's central insight to a case in which equity is already freely traded and therefore to the case of publicly traded corporations in the developed countries.

[4] Our analysis is applicable to other forms of debt, such as income bonds. It may also help to explain why income bonds failed their promise--see McConnell and Schlarbaum (1986). The variable to which the promised payment of an income bond is tied is not entirely exogenous. The income bond therefore retains the adverse selection property of outside equity sales.

this responsiveness to an exogenous signal of performance that a commodity price link establishes for the debt contract.[5,6]

Using the model constructed in section 1 we can directly measure the advantages of a commodity linked bond by calculating the agency costs of different debt instruments. We compare bonds with annual payments fixed in real terms, $\psi(s)\equiv\delta$, to bonds with annual payments tied to the realized real commodity price, $\psi(s)\equiv\theta s$. In Table 4 we displayed the equity owners' optimal operating strategy given a fixed rate bond for a hypothetical example, and in Table 3 we displayed the corresponding values of the firm, the equity, and the fixed rate bond. The comparable data for a commodity linked bond is displayed in Tables 5 & 6.

[Insert Tables 5 & 6]

The operating policy induced by the commodity linked bond is closer to the first best than the operating policy induced by the fixed rate bond. The value of the firm with the commodity linked bond is always greater than the value of the firm with the fixed rate bond. At a copper price of $0.60/pound the value of the commodity linked bond is marginally greater than the fixed rate bond; but the value of the firm and therefore of the equity levered with

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[5] In explaining the benefits of commodity linked financing, Gaylen Byker, head of Banque Paribas' Commodity-Indexed Transactions Group, put the agency case succinctly: "Nobody can say that he didn't meet a budget because the price of oil or some other commodity increased unexpectedly. There is something you can do about it" (Washington Post, July 26, 1989, D1).

[6] Since the realization of the commodity price is common knowledge it would seem that there exists an optimal financing contract in which the operating policy to be implemented is specified directly and the equity owners are declared in default when this policy is not followed. A commodity linked bond would then be an unnecessarily complicated incentive device. If, however, in addition to the commodity price there is some other variable determining the firm's profitability, and if this other variable is unobservable, then the commodity linked bond is the optimal contract. In Gale and Hellwig (1985) it is shown that a fixed rate debt contract is optimal when the firm's return is determined by a single unobservable variable: if it were determined by two variables, one unobservable and the other observable, then the analogous results would be that a debt contract with payments contingent upon the unobservable variable would be optimal. This is our commodity linked bond.

the commodity linked bond is significantly greater than the value of the firm and the equity levered with the fixed rate bond.

Given any fixed rate bond and given any current commodity price, it is always possible to identify a commodity linked bond that Pareto dominates the fixed rate bond as we now illustrate. We say that two bonds are bondholder-equivalent if the market values of the two bonds are equal given the current commodity price, $\hat{s}$, the current mine inventory, Q, and the current state of the mine, j: $b^{\delta}(\hat{s},Q;j) = b^{\theta}(\hat{s},Q;j)$. In this case the bondholder would be indifferent regarding a swap of one bond for the other. This equality implicitly defines a function relating the parameter of a given fixed rate bond to the parameter of its bondholder-equivalent commodity linked bond: $\theta \equiv \Theta^{b}(\delta;\hat{s},Q,j)$, iff $b^{\delta}(\hat{s},Q;j) = b^{\theta}(\hat{s},Q;j)$. Although the fixed rate and the commodity linked instruments are bondholder-equivalent, they are not equivalent for the equity holder nor for the firm as a whole: $e^{\delta}(\hat{s},Q;j) < e^{\Theta^{b}(\delta)}(\hat{s},Q;j)$ and $v^{\delta}(\hat{s},Q;j) < v^{\Theta^{b}(\delta)}(\hat{s},Q;j)$. The Modigliani-Miller result does not obtain. In Table 7 we list, for a range of current commodity prices, the commodity linked bond that is bondholder-equivalent to the original fixed rate bond. We also list the levered equity values given the fixed rate bond and given the commodity linked bond and the difference between the two. The difference between the value of the equity levered with the commodity linked bond and levered with the fixed rate bond is a direct measure of the relative agency costs of the two forms of debt financing.

[Insert Table 7]

An alternative expression of the superiority of commodity linked instruments is the increase in debt capacity they afford the firm. In Figure 1 we graph the value of the firm and the value of the fixed rate bond as a

function of the annual promised payment on the bond. Due to the sharply increasing agency costs, at some point increasing the promised debt payment causes the total value of the bond to fall. Consequently there exists a maximum value of debt that can be feasibly sold against the mine: we use this maximum as one measure of the fixed rate debt capacity of the mine. We have also graphed in Figure 1 the value of the firm and the value of the commodity linked bond as a function of the annual promised payment. One can see in the figure that the debt capacity of the mine is greater for commodity linked bonds than for fixed rate bonds. For our hypothetical mine, if the current commodity price is $ 0.55/pound, then the maximum value of fixed rate debt that could be sold is $ 8.80 million while the maximum value of commodity linked debt that could be sold is 9.01 million.

[Insert Figure 1]

3. An Analysis of Alternative Debt Designs

It is possible to derive closed form solutions for the value of the firm and for the optimal operating policy in the case that the inventory of the mine is infinite, $Q=\infty$. The differential equations governing the value of the equity when the inventory of the mine is infinite become:

$$\tfrac{1}{2}\sigma^2 s^2 e_{ss}(s;1) + (r-\kappa)s e_s(s;1) - m - \psi(s) - re(s;1) = 0, \tag{25}$$

for the closed mine, and

$$\tfrac{1}{2}\sigma^2 s^2 e_{ss}(s;2) + (r-\kappa)s e_s(s;2) + q(s-a) - \psi(s) - re(s;2) = 0, \tag{26}$$

for the open mine. The relevant boundary conditions are:

$$e_s(s_d^{\psi};1) = 0, \tag{27}$$

$$e_s(s_1^{\psi};2) = \begin{cases} e_s(s_1^{\psi};1) & \text{if } e(s_1^{\psi};1) - k_1 \geq 0 \\ 0 & \text{if } e(s_1^{\psi};1) - k_1 < 0, \end{cases} \tag{28}$$

$$e_s(s_2^\psi;1) = e_s(s_2^\psi;2), \tag{29}$$

$$e(s_d^\psi;1) = 0, \tag{30}$$

$$e(s_1^\psi;2) = \max\{e(s_1^\psi;1)-k_1,\ 0\}, \tag{31}$$

$$e(s_2^\psi;1) = e(s_2^\psi;2) - k_2. \tag{32}$$

The complete solutions to equations (25) and (26) are of the form

$$e(s;1) = \beta_1 s^{\gamma_1} + \beta_2 s^{\gamma_2} - m/r - \xi(\psi). \tag{33}$$

$$e(s;2) = \beta_3 s^{\gamma_1} + \beta_4 s^{\gamma_2} + qs/\kappa - qa/r - \xi(\psi), \tag{34}$$

where $\gamma_1 \equiv \alpha_1+\alpha_2$, $\gamma_2 \equiv \alpha_1-\alpha_2$, $\alpha_1 \equiv \frac{1}{2}-[(r-\kappa)/\sigma^2]$, and $\alpha_2 \equiv [\alpha_1^2 + 2r/\sigma^2]^{\frac{1}{2}}$. If we restrict our attention to bonds with payment functions of the form $\psi(s) = \delta + \theta s^n$, then $\xi(\psi)$ is obtained from the particular integral solution of the ordinary differential equation

$$v(s) = \sum_{i=0}^{z} A_i s^i, \qquad z = \max\{n, 2\}, \tag{35}$$
$$\Delta i = \begin{cases} n & n<1 \\ 1 & n\geq 1 \end{cases}$$

and,

$$\xi = \sum_{\ell=0}^{2} c_\ell [v_\ell(A_{j\neq i}=0] s^i \tag{36}$$

where c_ℓ is the coefficient of the ℓ^{th} derivative of v, v_ℓ, with respect to s. As in Brennan and Schwartz (1985), since $\gamma_1>1$ and we require that e/s remain finite as $s\to\infty$, it follows that $\beta_3=0$. The constants β_1, β_2, and β_4 as well as the optimal policy, $\phi'^\psi=(s_d^\psi, s_1^\psi, s_2^\psi)$ are determined by the boundary conditions (27)-(32) which imply:

$$\beta_1 = [fs_2(\gamma_2-1)+b\gamma_2]/(\gamma_2-\gamma_1)s_2^{\gamma_1} \tag{37}$$

$$\beta_2 = [\xi s_d^n(n-\gamma_1)-d\gamma_1]/(\gamma_2-\gamma_1)s_d^{\gamma_2} \tag{38}$$

$$\beta_4 = \beta_2 + [fs_1(\gamma_1-1)+g\gamma_1]/(\gamma_2-\gamma_1)s_1^{\gamma_2} \tag{39}$$

$$s_1 = x\gamma_1(g-bx^{\gamma_2})/f(x^{\gamma_2}-x)(\gamma_1-1) \tag{40}$$

$$s_2 = \gamma_2(g-bx^{\gamma_1})/f(x^{\gamma_1}-x)(\gamma_2-1) \tag{41}$$

where $f \equiv q/\kappa$, $d \equiv m/r$, $b \equiv -k_2-[(qa-m)/r]$, $g \equiv k_1-[(qa-m)/r]$, and where $x=s_1/s_2$, the ratio of the commodity prices at which the mine is closed and opened, and x is the solution to the non-linear equation

$$\frac{(x^{\gamma_2}-x)(\gamma_1-1)}{\gamma_1(g-bx^{\gamma_2})} = \frac{(x^{\gamma_1}-x)(\gamma_2-1)}{\gamma_2(g-bx^{\gamma_1})} . \tag{42}$$

The ratio $y=s_d/s_2$ is the solution to the non-linear equation

$$\frac{y^{\gamma_1}}{\xi s_d^n(\gamma_2-n)-\gamma_2(by^{\gamma_1}-d)} = \frac{x^{\gamma_1}-x}{\gamma_2(g-bx^{\gamma_1})} . \tag{43}$$

With these results in mind we are now ready to present a series of propositions regarding the nature of the optimal debt contract.

Proposition 1: *In the infinite inventory case the open and closure policies for any two debt contracts are identical. That is,* $\forall\ \psi,\psi'\quad (s_1^{\psi},s_2^{\psi}) = (s_1^{\psi'},s_2^{\psi'})$.

Proof: From equations (37)-(43) it is clear that the structure and parameters of the debt contract ψ enter into the solution of the solution for the optimal operating policy of the firm only through $\xi(\psi)$, and as one can see in equations (40)-(42), the values for s_1^{ψ} and s_2^{ψ} depend only upon the constants b, f, d, g, γ_1, γ_2, and not on $\xi(\psi)$.

Corollary 1.1: *The open and closure policies are identical under fixed rate and commodity linked debt:* $\forall\ \delta,\theta\quad (s_1^{\delta},s_2^{\delta}) = (s_1^{\theta},s_2^{\theta})$.

Corollary 1.2: *The first best open and closure policy is always implemented:* $\forall\ \psi\ \ (s_1^{\psi}, s_2^{\psi}) = (s_1^{FB}, s_2^{FB})$.

Remarks: (i) The intuition behind the proposition is as follows. In the infinite inventory case the open and closure decision is separable from the default decision. Since the inventory of the mine is infinite and unaffected by the open and closure decision, the rate of extraction from the mine does not affect the future payouts to the bondholder. Given any choice for a critical default price, the expected payout to the bondholder is fixed. The decision to open and close the mine therefore affects the marginal return to the equity holder exactly as it affects the marginal return to the bondholder, and therefore the equity holder will choose the first best open and closure policy. This is not true in the finite inventory case since the open and closure decision affects the total inventory of the mine which in turn affects whether the bond will be completely paid before the mine is exhausted.
(ii) It is important to note that in general the default decision of the firm is significantly affected by the type of debt and this means that the first best value of the firm will not be attained.

Proposition 2: *If the costs of financial distress are zero--or equivalently, if subsequent to declaration of default the firm would be operated according to the first best policy--then the value of the levered firm is equal to the first best.* $\alpha=1\ \Rightarrow\ \forall\ \psi, s, j\ \ v^{\psi}(s;j) = v^{FB}(s;j)$, *or equivalently,* $v(s_d;1) = v^{FB}(s_d;1)\ \ \Rightarrow\ \ \forall\ \psi, s, j\ \ v^{\psi}(s;j) = v^{FB}(s;j)$.

Proof: v^{ψ} solves the two differential equations

$$\tfrac{1}{2}\sigma^2 s^2 v_{ss}(s;1) + (r-\kappa)sv_s(s;1) - m - rv(s,Q;1) = 0, \tag{44}$$

and

$$\tfrac{1}{2}\sigma^2 s^2 v_{ss}(s;2) + (r-\kappa)sv_s(s;2) + q(s-a) - rv(s,Q;2) = 0, \tag{45}$$

subject to the boundary conditions:

$$v(s_d^{\psi};1) = \alpha\ v^{FB}(s_d^{\psi};1), \tag{46}$$

$$v(s_1^{\psi};2) = \max\{v(s_1^{\psi};1) - k_1, 0\}, \tag{47}$$

$$v(s_2^{\psi};1) = v(s_2^{\psi};2) - k_2. \tag{48}$$

By assumption of the proposition, condition (46) is satisfied for $v = v^{FB}$. By Corollary 1.2 it is the case that $(s_1^{\psi}, s_2^{\psi}) = (s_1^{FB}, s_2^{FB})$ and therefore conditions (47) and (48) can be rewritten as

$$v(s_1^{FB};2) = \max\{v(s_1^{FB};1) - k_1, 0\},$$

$$v(s_2^{FB};1) = v(s_2^{FB};2) - k_2,$$

which are both satisfied for $v = v^{FB}$ by definition. v^{FB} therefore satisfies (44) and (45) subject to (46)-(48) and so $v^{\psi} = v^{FB}$. □

Proposition 3: *If the debt requires no payments whenever the mine should be closed, then the first best operating policy is implemented: in particular the equity owner's optimal critical default price is equal to the first best abandonment price and the value of the levered mine is equal to the first best value of the mine. That is, if* ψ s.t. $\forall\ t_1, t_2$, $s(t_1) < s_1^{FB}$ and $\forall \tau \in (t_1, t_2)$ $s(\tau) < s_2^{FB}$, $\psi(s) \equiv 0$, *then* $(s_d^{\psi}, s_1^{\psi}, s_2^{\psi}) = (s_0^{FB}, s_1^{FB}, s_2^{FB})$ and $v^{\psi} = v^{FB}$.

Proof: The value of the first best mine and the parameters of the first best operating policy are given by a set of equations similar to (33)-(43) in which v^{FB} is substituted for e and the terms containing $\xi(s)$ are dropped: we denote by x^{ψ} and y^{ψ} and by x^{FB} and y^{FB} the relevant parameters given by equations (42)

and (43) for the levered and for the first best case, respectively. From equation (43) we can see that y^{ψ} and y^{FB} are determined identically by the constants b, d, g, γ_1 and γ_2 and by the values of x^{ψ} and x^{FB}, respectively: the term $\xi s_d^n(\gamma_2 - n)$ in equation (43) for the levered case will be zero by the conditions of the proposition, while this term simply does not appear in the first best case. It has already been established in Corollary 1.2 that $(s_1^{\psi}, s_2^{\psi}) = (s_1^{FB}, s_2^{FB})$, from which it follows that $x^{\psi} \equiv s_1^{\psi}/s_2^{\psi} = s_1^{FB}/s_2^{FB} \equiv x^{FB}$, and therefore that $y^{\psi} = y^{FB}$ and so $s_d^{\psi} = s_0^{FB}$. □

Remark: It has already been established that the open an closure decision is independent of the bond structure, so that the only new point is that this special bond structure induces the equity holder to make the first best default decision. The intuition for the result is very simple. If the bond requires no payment when the mine is closed, then a marginal variation in the critical default price does not change the expected payments to the bondholders: the equity holder bears the full marginal return to the default decision.

Proposition 4: *For every commodity linked bond, and for any current commodity price, there is a Pareto superior commodity linked bond: if* $\alpha < 1$, *then* $\forall\ \delta, s$ $\exists\ \theta$ s.t. $b^{\theta}(s;j) \geq b^{\delta}(s;j)$, $e^{\theta}(s;j) > e^{\delta}(s;j)$ and $v^{\theta}(s;j) > v^{\delta}(s;j)$.

Proof: First we establish that $\forall\ \delta$ $v_s^{\delta}(s) > \alpha v_s^{FB}(s)$. The equation describing the value of the levered firm is $v^{\delta}(s) = \beta_1^{\delta} s^{\gamma_1} + \beta_2^{\delta} s^{\gamma_2} - m/r$ and the equation describing the first best value of the firm is $v^{FB}(s) = \beta_1^{FB} s^{\gamma_1} + \beta_2^{FB} s^{\gamma_2} - m/r$. It can be easily verified that $\beta_1^{\delta} = \beta_1^{FB}$. At s_d^{δ} we have $v^{\delta}(s_d^{\delta}) \equiv \alpha v^{FB}(s_d^{\delta})$ and

therefore $\beta_1 s^{\gamma_1} + \beta_2^{\delta} s^{\gamma_2} - m/r = \alpha\{\beta_1 s^{\gamma_1} + \beta_2^{FB} s^{\gamma_2} - m/r\}$. Using this equality we can solve for $\beta_2^{\delta} = \alpha\beta_2^{FB} - (1-\alpha)\beta_1(s_d^{\delta})^{\gamma_1 - \gamma_2} + (1-\alpha)(m/r)(s_d^{\delta})^{-\gamma_2}$. Substituting back into the equation for $v^{\delta}(s)$ we have $v^{\delta}(s) - \alpha v^{FB}(s) = (1-\alpha)\{\beta^1[s^{\gamma_1} - (s_d^{\delta})^{\gamma_1}(s/s_d^{\delta})^{\gamma_2} - (m/r)(1-(s/s_d^{\delta})^{\gamma_2})\}$. Since by assumption $s > s_d^{\delta}$, and since $\gamma_1 > 0$ and $\gamma_2 < 0$ it must be the case that $v^{\delta}(s) - \alpha v^{FB}(s) > 0$ which completes the first step of the proof. Second, we establish that $\forall\ \delta\ \exists\ \delta' < \delta$ s.t. $b^{\delta'}(s_d^{\delta}) > b^{\delta}(s_d^{\delta})$.

$b^{\delta'}(s_d^{\delta}) - b^{\delta}(s_d^{\delta}) = [b^{\delta'}(s_d^{\delta}) - b^{\delta'}(s_d^{\delta'})] + [b^{\delta'}(s_d^{\delta'}) - b^{\delta}(s_d^{\delta})] =$
$b_s^{\delta}(s_d^{\delta'})(s_d^{\delta} - s_d^{\delta'}) + \epsilon_1 - \alpha v_s^{FB}(s_d^{\delta'})(s_d^{\delta} - s_d^{\delta'}) + \epsilon_2 = [b_s^{\delta}(s_d^{\delta'}) - \alpha v_s^{FB}(s_d^{\delta'})](s_d^{\delta} - s_d^{\delta'}) + \epsilon_1 + \epsilon_2 >$
$\epsilon_1 + \epsilon_2 \underset{\delta' \to \delta}{\to} 0$. Finally, we show that $\forall\ \delta'\ \exists\ \theta$ s.t. $\forall\ s > s_d^{\delta'}$ $b^{\theta}(s) > b^{\delta'}(s)$. Define $\theta \equiv \Theta^e(\delta;s)$ implicitly s.t. $e^{\Theta^e(\delta;s)}(s) = e^{\delta}(s)$. It can be shown that Θ is increasing in s, or alternatively that $\forall\ s > s_d^{\delta}$ $e^{\Theta^e(\delta;s_d^{\delta})}(s) < e^{\delta}(s)$. Since by definition $s_d^{\Theta^e(\delta;s_d^{\delta})} = s_d^{\delta}$ and therefore $\forall\ s$ $v^{\Theta^e(\delta;s_d^{\delta})}(s) = v^{\delta}(s)$ it also follows that $b^{\Theta^e(\delta;s_d^{\delta})}(s) > b^{\delta}(s)$. These three steps establish that $\forall\ \delta\ \exists\ \theta$ s.t. $\forall$ s $b^{\theta}(s) \geq b^{\delta}(s)$ and $v^{\theta}(s;j) > v^{\delta}(s;j)$. By continuity of s_d^{θ} and v^{θ}, e^{θ}, and b^{θ} in θ it is a simple matter to choose θ conditional on s so that the conditions of the proposition obtain. $\square$

Remark: The intuition for the proof is simplest in the case that the firm is worthless upon default. Assume that the firm has outstanding some fixed rate bond and that the price of the commodity has fallen to the critical default price. The firm is about to declare defualt and experience a deadweight loss: the outstanding bond is worthless regardless of the promised payments. It is clearly possible to negotiate a lower promised payment such that the firm will not default at the current price, and a bond with this lower promised payment will have a positive value, a value greater than a bond with the higher

promised payment. In this extreme case in which the commodity price has fallen to the critical default value, the marginal benefit created by increasing the total value of the firm is clearly greater than the marginal loss in coupon payments should the commodity price once again rise. When the commodity price is not equal to the critical default value, this tradeoff may go either way, depending upon the parameter of the outstanding bond. However, this same tradeoff is always positive when considering a marginal drop in the fixed payment in exchange for a marginal increase in the commodity linked payment. Shifting to a commodity linked payment clearly lowers the coupons should the price fall, and therefore lowers the critical default price and increases the total value of the firm. Moreover, since the value of the bond is increasing in the commodity price itself, there is no corresponding loss in the event of a future rise in the commodity price.

4. Conclusion

In this paper we present three results. First, we show how to adapt the traditional contingent claims valuation techniques to correctly value the firm and its liabilities in the presence of agency costs. Second, we can then measure the significance of the agency costs as a function of the quantity of debt outstanding and as a function of the design of the debt contract: with this we determine the relative benefits of alternative contract designs. Third, we apply this technique to the case of commodity linked bonds: while previous models could not be used to explain which firms should issue this new instrument, nor why this innovation has arisen, we provide an answer to both of these questions. Moreover, while in the traditional agency literature only qualitative insights are offered, our model can be used in practice.

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Table 1

Data for the Hypothetical Firm

Total inventory in the ground: $Q = 150$ million pounds
Annual production for an open mine: $q = 10$ million pounds
Average real production costs: $a = \$0.50$ per pound
Maintenance costs for a closed mine: $m = \$\ 0$
Real opening and closing cost: $k_1 = k_2 = \$2$ million
Real interest rate: $r = 2\%$
Commodity price variance: $\sigma = 8\%$
Convenience yield: $\kappa = 1.5\%$

Table 2

<u>The Levered Firm's Operating Policy--Fixed Rate Bond</u>

Annual real coupon payments: δ = \$0.4 million
Factor of firm's first best value at bankruptcy: $\alpha = 0$

critical commodity prices (\$/pound)	First Best Operating Policy	Equity Owners' Optimal Operating Policy
abandonment/default	$s_0 = 0.00$	$s_d = 0.40$
closing	$s_1 = 0.59$	$s_1 = 0.54$
opening	$s_2 = 0.84$	$s_2 = 0.79$

Table 3

<u>The Value of the Levered Firm--Fixed Rate Bond</u>

Annual real coupon payments: δ = \$0.4 million
Factor of firm's first best value at bankruptcy: $\alpha = 0$

commodity price s	Firm Value $v^{\flat}(s,Q;j)$ (closed) j=1	Firm Value $v^{\flat}(s,Q;j)$ (open) j=2	Equity Value $e^{\flat}(s,Q;j)$ (closed) j=1	Equity Value $e^{\flat}(s,Q;j)$ (open) j=2	Bond Value $b^{\flat}(s,Q;j)$ (closed) j=1	Bond Value $b^{\flat}(s,Q;j)$ (open) j=2
0.05	0.00		0.00		0.00	
0.10	0.00		0.00		0.00	
0.15	0.00		0.00		0.00	
0.20	0.00		0.00		0.00	
0.25	0.00		0.00		0.00	
0.30	0.00		0.00		0.00	
0.35	0.00		0.00		0.00	
0.40	2.27		0.14		2.13	
0.45	6.13		1.85		4.28	
0.50	10.13		4.76		5.37	
0.55	14.34	11.36	8.55	6.74	5.79	4.62
0.60	18.90	16.44	13.00	12.15	5.90	4.27
0.65	23.78	22.85	18.00	18.18	5.78	4.67
0.70	29.06	29.47	23.47	24.55	5.59	4.92
0.75	34.75	36.12	29.39	31.10	5.36	5.02
0.80		42.84		37.74		5.10
0.85		49.60		44.44		5.16
0.90		56.37		51.17		5.20
0.95		63.13		57.91		5.22
1.00		69.91		64.68		5.23

Table 4

The Agency Cost of Debt--Fixed Rate Bond

Annual real coupon payments: δ = \$0.4 million
Factor of firm's first best value at bankruptcy: α = 0

commodity price s	Firm Value				Difference			
	first best $v^{FB}(s,Q;j)$		levered $v^{L}(s,Q;j)$		absolute value $v^{FB} - v^{L}$		percent of first best $(v^{FB} - v^{L})/v^{FB}$	
	(closed) j=1	(open) j=2	(closed) j=1	(open) j=2				
0.05	0.00		0.00		0.00		--	
0.10	0.08		0.00		0.08		100	
0.15	0.38		0.00		0.38		100	
0.20	1.04		0.00		1.04		100	
0.25	2.10		0.00		2.10		100	
0.30	3.57		0.00		3.57		100	
0.35	5.45		0.00		5.45		100	
0.40	7.75		2.27		5.48		70.7	
0.45	10.46		6.13		4.33		41.4	
0.50	13.58		10.13		3.45		25.4	
0.55	17.14		14.34	11.36	2.80	3.78	16.3	25.0
0.60	21.12	19.20	18.90	16.44	2.22	2.76	10.5	14.4
0.65	25.54	24.46	23.78	22.85	1.76	1.61	6.9	6.6
0.70	30.41	30.36	29.06	29.47	1.35	0.89	4.4	2.9
0.75	35.73	36.64	34.75	36.12	0.98	0.52	2.7	1.4
0.80	41.52	43.14		42.84	0.67	0.30	1.6	0.7
0.85		49.76		49.60		0.16		0.3
0.90		56.45		56.37		0.08		0.1
0.95		63.18		63.13		0.05		0.1
1.00		69.93		69.91		0.02		0.0

Table 5

The Levered Firm's Operating Policy--Commodity Linked Bond

Annual real coupon payments: θs = \$0.06 s million
Factor of firm's first best value at bankruptcy: $\alpha = 0$

critical commodity prices (\$/pound)	Equity Owners' Optimal Operating Policy
abandonment/default	$s_d = 0.32$
closing	$s_1 = 0.55$
opening	$s_2 = 0.80$

Table 6

<u>The Value of the Levered Firm--Commodity Linked Bond</u>

Annual real coupon payments: θs = \$0.06 s million
Factor of firm's first best value at bankruptcy: α = 0

commodity price s	Firm Value $v^{\yen}(s,Q;j)$ (closed) j=1	(open) j=2	Equity Value $e^{\yen}(s,Q;j)$ (closed) j=1	(open) j=2	Bond Value $b^{\yen}(s,Q;j)$ (closed) j=1	(open) j=2
0.05	0.00		0.00		0.00	
0.10	0.00		0.00		0.00	
0.15	0.00		0.00		0.00	
0.20	0.00		0.00		0.00	
0.25	0.00		0.00		0.00	
0.30	0.00		0.00		0.00	
0.35	1.62		0.27		1.35	
0.40	4.71		1.39		3.32	
0.45	8.01		3.49		4.52	
0.50	11.53		6.36		5.17	
0.55	15.28		9.87		5.41	
0.60	19.69	17.52	13.96	12.72	5.73	4.80
0.65	24.28	23.27	18.53	18.40	5.75	4.87
0.70	29.42	29.47	23.57	24.43	5.86	5.04
0.75	34.95	36.16	29.03	30.64	5.92	5.52
0.80		42.87		36.93		5.94
0.85		49.62		43.26		6.36
0.90		56.37		49.63		6.74
0.95		63.14		56.01		7.13
1.00		69.91		62.39		7.52

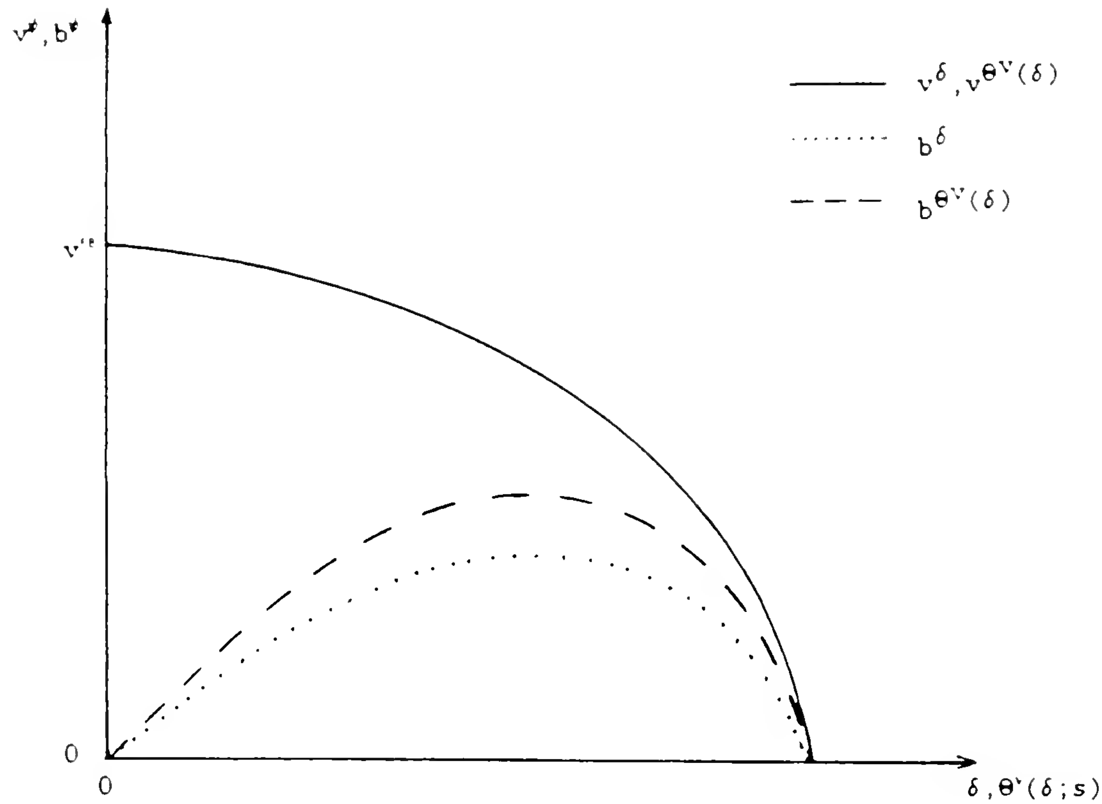
Table 7

Pareto Superior Commodity Linked Bonds

current commodity price	fixed rate bond value	commodity link	commodity linked bond value	Equity Value fixed rate financed	Equity Value commodity link financed
$\hat{s}$	b^{δ}	$\theta^{b}(\delta;\hat{s})$	$b^{\theta^{b}(\delta)}$	e^{δ}	$e^{\theta^{b}(\delta)}$
0.40	2.13	.017	2.17	0.14	5.43
0.45	4.28	.040	4.29	1.85	5.08
0.50	5.37	.065	5.38	4.76	5.94
0.55	5.79	.068	5.81	8.55	9.17
0.60	5.90	.065	5.95	13.00	13.49
0.65	5.78	.061	5.83	18.00	18.44
0.70	5.59	.056	5.61	23.47	23.96
0.75	5.36	.053	5.44	29.39	29.74
0.80	5.10	.052	5.16	37.74	37.74
0.85	5.16	.049	5.17	44.44	44.45
0.90	5.20	.046	5.21	51.17	51.17
0.95	5.22	.044	5.22	57.92	57.92
1.00	5.23	.042	5.23	64.68	64.68

Figure 1

Debt Capacity with Fixed Rate and Commodity Linked Bonds



The solid line plots the value of the firm levered with a fixed rate bond at a given commodity price as the coupon payment on the debt is increased from zero. The same line plots the value of the firm levered with commodity linked debt at the given price as the commodity link parameter is increased from zero at a comparable rate: $\theta^{v}(\delta; s) \Rightarrow v^{\theta}(s) - v^{\delta}(s)$. The dotted line plots the value of the fixed rate bond at a given commodity price as the coupon payment on the debt is increased from zero. The dashed line plots the value of the commodity linked bond at a given commodity price as the commodity linked parameter on the debt is increased from zero at the comparable rate. Notice that the total value of the fixed rate bond has a global maximum, and that this global maximum lies below that for the commodity linked bond. Moreover, for any value of the fixed rate bond, there is a commodity linked bond with an equivalent value but which yields a higher total value for the firm.